Instabilities of the Magnetic Field in a Neutron Star

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1. Aims/Objectives

To see how the magnetic field in a neutron star behaves:

- Reproduce the work of Wood *et al*[1]
- Analyse the linear stability of equilibria using equation (2)
- Simulate the evolution of a magnetic field using numerical code.

2. Equations

The magnetic field, B, evolves according to the following equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\frac{c}{4\pi e n} \mathbf{B} \times (\nabla \times \mathbf{B}) \right) - \nabla \times (\eta \nabla \times \mathbf{B}) \quad (1)$$

where c is the speed of light, e=|e| is the elementary charge, n is the electron number density, and η is the magnetic diffusivity.

The stabiliity of an equilibrium magnetic field, B, can be determined by the linear equation:

$$\frac{\partial}{\partial t}\delta \mathbf{B} = \nabla \times \left[-\frac{1}{n}\mathbf{J}_0 \times \delta \mathbf{B} + \frac{1}{n}\mathbf{B}_0 \times (\nabla \times \delta \mathbf{B}) \right] \quad (2)$$

3. Methods

A pseudo-spectral code adapted from the 3D MHD code PARODY developed by Dormy *et al*[2] and Aubert *et al*[3] was used to solve equation (1). All simulations were performed in Fortran and analysed in MATLAB.

4. Results

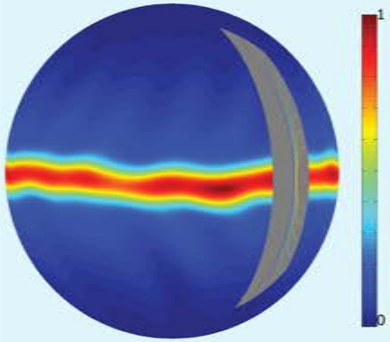

Figure 1: Plot showing the ratio of the Maxwell stress to the yield stress of the star's crust, i.e. the likelihood of a starquake, and the lines of the polodial axisymmetric field in a meridional cross-section

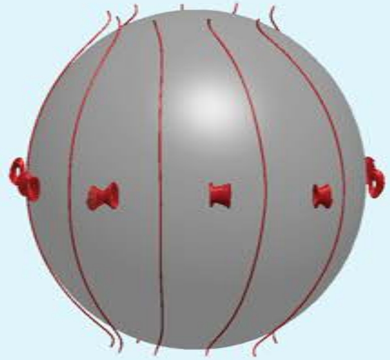

Figure 2: 3D plot of the magnetic field lines. The grey spherical surface indicates the bottom of the crust where the field lines do not cross due to the superconducting nature of the core.

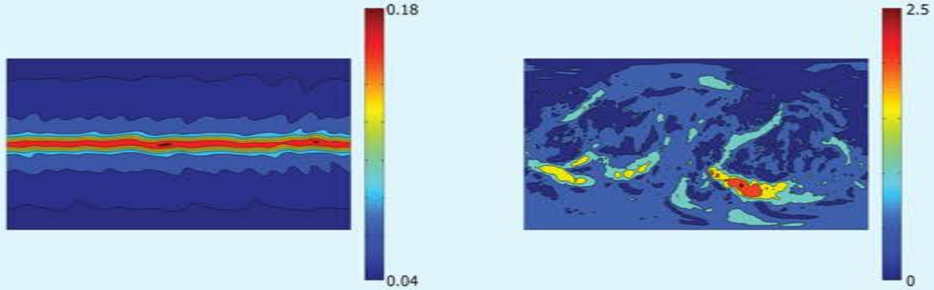

Figure 3: Surface projections of the magnetic field strength for two different simulations. The magnetic field is strongest around the equator of the neutron star in the first plot, whereas the second plot is more spatially disordered, with patches of strong field.

5. Conclusions

- Several plots have been produced that illustrate the magnetic field in a neutron star can change a lot depending on the initial conditions for the magnetic field.
- Through extending previously published results, it has been shown that a magnetic instability can occur under the conditions present in neutron stars.
- For a magnetic field of the form $B(z) = 1 - tanh(z)$ and electron density $n(z) = 1 - tanh(z)$ the most unstable mode has growthrate $\lambda = \frac{3-\sqrt{5}}{4}$

6. Acknowledgements

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7. References

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